(AP) Kentucky's Affordable Prepaid Tuition

# Annual Actuarial Valuation of the Prepaid Tuition Trust Fund For Kentucky's Affordable Prepaid Tuition 

June 30, 2006

Prepared by
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September 27, 2006

Ms. Jo Carole Ellis
Director
Kentucky's Affordable Prepaid Tuition
100 Airport Road, P.O. Box 798
Frankfort, KY 40601

Dear Ms. Ellis:
We have completed our actuarial analysis of the Prepaid Tuition Trust Fund ("the Fund") for Kentucky's Affordable Prepaid Tuition ("KAPT" or "the Program") as of June 30, 2006. This report presents our findings with respect to the Fund's expected cash flows and adequacy of the Fund in light of assets in the Fund.

The analysis of the funding of the Program was prepared for the KAPT Board for the purpose of assessing the actuarial soundness of the Fund as required by statute. The analyses have been prepared in accordance with generally accepted actuarial principles and practices commonly applicable to similar types of arrangements.

Currently the expected value of liabilities is $\$ 164,072,370$ and the value of assets is $\$ 143,763,132$ for a difference of $(\$ 20,309,238)$ or $12.4 \%$ of liabilities. These results are based on assumptions approved by KAPT personnel after consultation with us.

We appreciate the opportunity to serve the Commonwealth of Kentucky. Any questions about the report should be directed to me at (770) 752-5656.

Very truly yours,


Robert B. Crompton, FSA, MAAA

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## I. EXECUTIVE SUMMARY

The following are the key findings of our analysis.

## Status of the Program

The KAPT Fund's liabilities exceed its assets by $\$ 20,309,238$ resulting in a deficit. This result is based on the assumption that the Program will not sell any additional contributions.

The deficit is offset by the availability of the Kentucky Abandoned Property Fund as provided in KRS 393.015. Seventy-five percent of the Abandoned Property Fund is available for any unfunded liability of KAPT pertaining to contracts entered into before March 20, 2005. As of June 30, 2006, the balance of the Abandoned Property Fund is \$258,816,103.

If the Program continues to sell appropriately priced contracts, then the deficit is projected to be cured in as little as four years, depending on the number of contracts sold. This issue is addressed more fully in the Effects of Future Contract Sales section of this report.

Furthermore we note that the results above are based on a single baseline estimate of future experience. When potential volatility is considered, the Program is projected to have a $51 \%$ likelihood of at least breaking even. This issue is addressed more fully in the Monte Carlo Modeling section of this report.

The table following summarizes results for June 30, 2006:

| Value as of <br> June 30, 2006 | Assets and <br> Liabilities |
| :--- | ---: |
| Invested Assets \& Contract Receivables | $\$ 143,050,994$ |
| Other Receivables \& Accruals | $\$ 712,138$ |
| Actuarial Liabilities | $\$ 164,057,395$ |
| Other Liabilities | $\$ \quad 14,975$ |
| Actuarial Deficit | $\$ 20,309,238)$ |
| Deficit as a Percent of Liabilities | $12.4 \%$ |

## Key Assumptions

Key economic assumptions are listed below.

| Key Assumptions |  |
| :--- | :--- |
| Yield on Investments |  |
| $\quad$ All Years | $7.76 \%$ |
| Investment returns are before expenses. |  |


| Key Assumptions (Continued...) |  |
| :--- | :--- |
| Tuition Inflation |  |
| All Classes of Contracts | $11.0 \%$ |
| $2007 / 08$ | $10.0 \%$ |
| $2008 / 09$ | $7.00 \%$ |
| All years thereafter | $\$ 647,397$ |
| Expenses |  |
| Initial Expenses |  |
| The initial expense is projected to |  |
| $\quad$ decrease over time as more contracts |  |
| are sold. |  |

The tuition inflation assumptions are based on a combination of statistical models of tuition increases and on actuarial judgment. Our statistical models use information from the past 20 years.

## II. RELIANCES \& COMPLIANCE WITH ACTUARIAL STANDARDS OF PRACTICE

In making the projections on which this report is based, we relied on the following information supplied to me as indicated below.

- Tuition amounts at Kentucky colleges and universities, public and private, supplied by the staff of KAPT
- Program expenses, supplied by the staff of KAPT
- Market value of assets of the Program's trust fund, supplied by the staff of KAPT
- Inventory of KAPT contracts by category, enrollment period, payment method and anticipated matriculation year, supplied by the Program's records administrator, Intuition Solutions, Inc.
- Assumptions regarding future investment returns on the Program's trust fund, supplied by the Program's investment advisor, Evaluation Associates
- Assumptions regarding the Program's anticipated asset allocation, supplied by the Program's investment advisor, Evaluation Associates

There are no actuarial standards of practice that apply specifically to prepaid tuition programs. However, there are two general standards that we believe apply:

- Actuarial Standard of Practice \#3 "Actuarial Communications". This standard sets general guidelines for actuarial communications. This report is in compliance with Standard \#3.
- Actuarial Standard of Practice \#23 "Data Quality". This standard sets guidelines on review of data supplied by a third party. We have performed reasonableness and consistency checks on the data supplied to us by personnel of the Program and by the records administrator, and are in compliance with this standard. Our review of the data was not an audit of the data.


## III. DESCRIPTION OF THE PROGRAM

The Program was created in 2000 by the Kentucky Legislature "to provide access to participating institutions for the qualified beneficiaries and to provide students and their parents' economic protection against rising tuition costs." The Legislature created the Prepaid Tuition Trust Fund in the custody of the state treasurer for administration by a board of directors. "The fund shall consist of payments received from prepaid tuition contracts. Income earned from the investments of the fund shall remain in the fund and be credited to it."

Administration of the Program and board governance now resides with the Kentucky Higher Education Assistance Authority.

## Description of Contracts $\mathcal{E}$ Payment Options

There are three types of contracts.

- The Value Plan, which provides in-state tuition at community colleges and technical colleges. Purchasers have the option of buying one year or two years of tuition under the Value Plan.
- The Standard Plan, which provides in-state tuition at any of Kentucky's eight public universities. The price for Standard Plan contracts is based on the most expensive public university. Purchasers have the option of buying from one year's tuition to five years' tuition in one-year increments.
- The Premium Plan, which is designed to cover the cost of average tuition at Kentucky's private colleges and universities. The cost of the Premium Plan contracts is based on the enrollment weighted-average tuition of Kentucky's private colleges and universities and increases at the same rate as tuition increases at the University of Kentucky. Similar to the Standard Plan, purchasers may purchase one year's tuition to five years' tuition in one-year increments.

Contracts are available to students who are at least two years away from initial college enrollment. Benefits can be used at any institution of higher education that is accredited by the U.S. Department of Education anywhere in the country. Benefits paid for out-of-state institutions or graduate schools will not exceed the benefits provided for Kentucky undergraduate benefits described above.

Each contract type has three main types of payment options:

- Lump Sum Payment
- Installment Payments, which come in several varieties:
o Monthly payments over three years
o Monthly payments over five years
o Monthly payments over seven years
o Monthly payments until the beneficiary's projected year of enrollment
- A combination of a Lump Sum down payment plus Installment Payments, where the installment payments are available in the following options:
o Monthly payments over three years
o Monthly payments over five years
o Monthly payments over seven years


## Residency Requirements

There are no residency requirements imposed on the purchasers of KAPT contracts.
KAPT beneficiaries can be either:

- Kentucky residents at the time the application is signed or
- Intend to attend college in Kentucky.


## Refunds

For cancellations other than death, disability, or receipt of a scholarship, the purchaser receives a refund of payments minus administrative charges and cancellation fees if the cancellation occurs before July 1 of the projected year of initial college enrollment. Cancellations for reasons other than death, disability, or receipt of a scholarship that occur on or after July 1 of the projected year of initial college enrollment will receive the tuition payout value of the contract minus administrative and cancellation fees.

If the beneficiary dies, becomes disabled, or receives a scholarship, the purchaser will receive a refund as described immediately above but with no deduction of any administrative or cancellation fees.

## Change of Beneficiary

A contract owner may request a change of beneficiary to a substitute who is a family member of the immediately-preceding beneficiary. Changes in beneficiary for reasons other than death, disability, or receipt of a scholarship of the original beneficiary will be subject to administrative fees.

## IV. SUMMARY OF CONTRACT DATA AND CURRENT ASSETS

## Contract Data

Data on the number of outstanding contracts and payments was provided by the Program's records administrator, Intuition Solutions, Inc. The graphs below summarize the data provided concerning these KAPT contracts.

Distribution of KAPT Contracts by Contract Type


## Distribution of KAPT Contracts by Year of Initial College Enrollment



## Current Assets

The assets currently held by the Fund are an important part of the determination of the actuarial adequacy of the Program. The investment strategy for those assets is also critical to the yield and to the vulnerability of the Program's actuarial adequacy to changes in the return earned on investments.

## Fund Investments

The total market value of assets held as of June 30, 2006 is $\$ 118,551,237$. The allocation of these assets is shown in the table below.

| Market value of cash \& invested assets held as of June 30, 2006 |  |  |
| :--- | ---: | ---: |
| Cash | $\frac{\text { Amount }}{483,211}$ | $\frac{\% \text { Of Total }}{0.41 \%}$ |
| Corporate Bonds | $15,375,045$ | $12.97 \%$ |
| U.S. Treasury and Government Agency Securities | $25,838,956$ | $21.80 \%$ |
| Corporate Stock | $68,641,586$ | $57.90 \%$ |
| Money Market Securities | $\underline{8,212,439}$ | $\underline{6.93 \%}$ |
| TOTAL | $\$ 118,551,237$ | $\underline{\underline{100.00 \%}}$ |

## Investment Strategy

The investment strategy is designed to achieve an investment return in excess of tuition inflation, which will allow KAPT to provide the contractual benefits to KAPT beneficiaries at their anticipated initial year of college enrollment. The Fund's asset allocation has a target allocation by asset category as follows:

- Large Cap U.S. Stocks

$$
45 \%
$$

- Small/Mid Cap U.S. Stocks 10\%
- Non-U.S. Stocks 5\%
- Inflation Indexed Bonds $25 \%$
- Corporate Bonds 15\%

We note that the current asset allocation is within the ranges allowed by the Program's Investment Policy.

## V. ACTUARIAL METHODS AND ASSUMPTIONS

## Methods

The actuarial method for the determination of the adequacy of the Fund consists of projecting future tuition rates, future expenses based on the average anticipated number of KAPT Contracts in place, and future utilization of KAPT Contracts. Future benefits and expenses are discounted using the assumed investment yield as the interest discount rate. The assumed discount rate is based on the current and anticipated mix of assets of the Fund.

For the projection of future benefits, the analysis proceeds as follows:

- Project future tuition rates for all years under consideration. Future tuition is based on the assumptions for tuition inflation. These assumptions vary by postsecondary school.
- Determine the nominal cost of future use of KAPT contracts based on the assumptions regarding utilization of contracts and the length of time the average beneficiary will take to complete his college education.
- Determine the nominal value of administrative expenses.
- Determine the present value of future contract usage and future expenses based on the investment yield assumptions.
- Perform projections for all of the Program's beneficiaries to determine if the Fund is adequate in the aggregate and make sufficient provision for overhead expenses.


## Assumptions

Actuarial assumptions used to determine financial soundness of programs are of two general types: economic and demographic. Demographic assumptions determine the expected exposure to financial claims and generally answer the question "How and when will people use their contract?" Economic assumptions are concerned with the expected level of contract usage and answer the question "What is the expected value of contract usage?" The assumptions that we used were those that were approved by the KAPT Director, after consultation with us.

## Economic Assumptions

Economic assumptions are used to estimate the annual tuition rates at two and four year colleges, increases in Fund expenses, and Fund earnings on assets invested. Because inflation is a major component of the rate of increase in tuition rates and of investment returns, we considered these rates together. We believe that the difference in these rates is more important than the absolute level of the rates. The following paragraphs describe the economic assumptions used in this study.

## Federal Income Tax

We assumed that Fund earnings are exempt from Federal Income Tax.

## Annual Tuition Rates

Tuition increases vary by duration and are shown in the table below. Our assumptions were guided by our observations of historic tuition increases, trends in postsecondary enrollment in Kentucky, and the level of legislative appropriations for postsecondary schools in Kentucky.

| Tuition Inflation |  |
| :---: | :---: |
| All Classes of Contracts |  |
| $2007 / 08$ | $11.0 \%$ |
| $2008 / 09$ | $10.0 \%$ |
| All years thereafter | $7.0 \%$ |

## Fund Earnings Rate

Our assumption for investment returns is based on information supplied to us by the Program's investment advisor, Evaluation Associates. Evaluation Associates supplied us with expected asset class returns. The assumption below is gross before expenses and is based on the asset class returns combined with the Program's target allocation ratios.

| Investment Returns |  |
| :---: | :---: |
| Investment Return for all future years | $7.76 \%$ |

## Annual Expenses

We are projecting future expenses to be as shown in the following table.

| Expenses |  |
| :---: | :---: |
| Investment Expenses | $0.35 \%$ |
| Applicable to all assets | $\$ 647,397$ |
| Administrative Expenses |  |
| Initial Annual Amount |  |
| This amount is assumed to decline |  |
| as |  |
| the Program grows. |  |

## Demographic Assumptions

The demographic assumptions used in this report are based on our experience with similar types of liabilities. Our choice of assumptions is based on recent experience and our best estimates as to future events. These assumptions are as follows:

## Contract Cancellations Due To Mortality and Disability

We assumed no contract terminations due to death or disability.

## Other Contract Cancellations

We assumed that contracts would cancel according to the tables given below.

|  | Contract Cancellation Table 1 of 2 |  |  |
| :--- | :---: | :---: | :---: |
|  |  | 36 Monthly | 60 Monthly |
| Type of Payment=> | Lump Sum | Payments | Payments |
| Year of purchase | $1.50 \%$ | $3.00 \%$ | $5.00 \%$ |
| Year of purchase+1 | $1.00 \%$ | $2.00 \%$ | $4.00 \%$ |
| Year of purchase+2 | $0.75 \%$ | $1.00 \%$ | $3.00 \%$ |
| Year of purchase+3 | $0.75 \%$ | $1.00 \%$ | $2.00 \%$ |
| Year of purchase+4 | $0.50 \%$ | $0.75 \%$ | $1.00 \%$ |
| Thereafter | $0.50 \%$ | $0.75 \%$ | $0.75 \%$ |


|  | Contract Cancellation Table 2 of 2 |  |  |
| :--- | :---: | :---: | :---: |
|  | 84 Monthly | Extended | Custom |
| Type of Payment=> | Payments | Payments | Payments |
| Year of purchase | $6.00 \%$ | $8.00 \%$ | $8.00 \%$ |
| Year of purchase+1 | $4.00 \%$ | $7.00 \%$ | $7.00 \%$ |
| Year of purchase+2 | $3.00 \%$ | $5.00 \%$ | $5.00 \%$ |
| Year of purchase+3 | $2.00 \%$ | $4.00 \%$ | $4.00 \%$ |
| Year of purchase+4 | $1.00 \%$ | $3.00 \%$ | $3.00 \%$ |
| Year of purchase+5 | $1.00 \%$ | $2.00 \%$ | $2.00 \%$ |
| Year of purchase+6 | $1.00 \%$ | $1.00 \%$ | $1.00 \%$ |
| Thereafter | $0.75 \%$ | $0.75 \%$ | $0.75 \%$ |

## Matriculation Percent

All beneficiaries are assumed to matriculate at the matriculation date specified in the application, except for those who are projected to terminate, die, or become disabled.

## Utilization of Benefits

We assume that beneficiaries will enroll in college at the date indicated as their anticipated matriculation date. We also assume that beneficiaries will use one year's worth of benefits over the course of only one academic year. That is, a 4-year contract will use all benefits over four academic years.

Within an academic year, contract usage is assumed to be $50 \%$ for the fall semester, $50 \%$ for the spring semester and none for the summer semester.

We believe these assumptions are slightly conservative since the alternate assumption is to assume that beneficiaries use their benefits more slowly. This slowdown in utilization would be beneficial to the Program since the anticipated Fund earnings rate will exceed the tuition increase rate after the first five years of the projection.

## Dropout Rate

All beneficiaries are assumed to use $100 \%$ of their contractual benefits once they have enrolled in college.

## Frequency of Beneficiary Replacement

Since all surviving beneficiaries are expected to matriculate and are expected to use their KAPT contracts until completion, the assumption is made that no replacement of beneficiaries will occur.

## VI. STATUS OF THE FUND AS OF JUNE 30, 2006

In determining the status of the Fund, we estimated the future disbursements for higher education expenses of beneficiaries, expenses, and refunds for terminated contracts. We also projected the future assets based on current assets and expected earnings on assets. We believe these estimates are reasonable based on the information available and our past experience and judgment.

The estimates of the prospective assets and liabilities of the Fund are summarized in the table on the following page and demonstrate the financial position of the Fund. The value of all assets is $\$ 143,763,132$ while the expected value of the actuarial liabilities is $\$ 164,057,395$. The resulting actuarial deficit is $\$ 20,309,238$.

The actuarial deficit will change from year to year due to positive and negative cash flows and due to the change in the present value of future contract usage and expense payments because of the passage of time. The actuarial deficit will also change due to the variance of experience from the assumptions. These variances include tuition increases, investment income, and expenses.

The deficit will also change due to the growth of the program and due to the updating of the assumptions to reflect the Program's emerging experience. The changes for the year ending June 30, 2006 are summarized in the table below.

| Progression of Deficit |  |  |
| :--- | ---: | ---: |
| Deficit at June 30, 2005 | $\left(\begin{array}{l}\text { ( }\end{array}\right.$ | $6,623,928)$ |
| Projected Increase to June 30, $2006^{1}$ | $(6,061,647)$ |  |
| Loss due to Unfavorable Tuition Inflation | $(1,317,903)$ |  |
| Loss due to Unfavorable Investment Experience | $-0-$ |  |
| Gain due to Additional Contract Sales | $(5,806,459)$ |  |
| Changes due to Change In Assumptions | $(341,342)$ |  |
| All Other Changes ${ }^{2}$ | $(\$ 20,309,238)$ |  |

[^0]In the following chart we show the value of expected future contract usage, expected future payments, current assets, and expected deficit as of the end of each future year for active contracts as of June 30,2006 . We note that the Fund is projected to have sufficient money to pay benefits until Fiscal 2021 - that is, for a period of 14 years.

PRESENT VALUE OF ASSETS AND LIABILITIES

| Fiscal Year Ending | Assets Other Than Future Revenues | Actuarial Value Of Future Revenues | Value of Liabilities | Actuarial Deficit |
| :---: | :---: | :---: | :---: | :---: |
| 2006 | 119,263,375 | 24,499,757 | 164,072,370 | $(20,309,238)$ |
| 2007 | 128,003,103 | 17,762,155 | 169,028,835 | $(23,263,578)$ |
| 2008 | 133,809,422 | 12,656,595 | 172,090,064 | $(25,624,047)$ |
| 2009 | 135,642,159 | 8,867,818 | 172,152,359 | $(27,642,382)$ |
| 2010 | 133,453,453 | 6,315,703 | 169,355,481 | $(29,586,325)$ |
| 2011 | 128,217,862 | 4,580,010 | 164,383,796 | $(31,585,924)$ |
| 2012 | 120,364,816 | 3,293,089 | 157,322,123 | $(33,664,217)$ |
| 2013 | 110,533,548 | 2,437,547 | 148,915,149 | $(35,944,055)$ |
| 2014 | 99,693,013 | 1,762,564 | 139,825,719 | $(38,370,142)$ |
| 2015 | 87,637,662 | 1,236,375 | 129,853,663 | $(40,979,626)$ |
| 2016 | 74,625,285 | 830,913 | 119,261,144 | $(43,804,946)$ |
| 2017 | 60,507,653 | 519,309 | 107,898,273 | $(46,871,310)$ |
| 2018 | 44,831,121 | 293,104 | 95,344,635 | $(50,220,410)$ |
| 2019 | 26,829,607 | 149,254 | 80,882,904 | $(53,904,043)$ |
| 2020 | 7,151,832 | 57,141 | 65,137,407 | $(57,928,435)$ |
| 2021 | $(13,881,563)$ | 15,582 | 48,476,186 | $(62,342,168)$ |
| 2022 | $(35,164,596)$ | 2,537 | 31,988,988 | $(67,151,046)$ |
| 2023 | $(53,563,551)$ | - 0 - | 18,792,112 | (72,355,663) |
| 2024 | $(68,713,900)$ | - 0 - | 9,256,563 | $(77,970,463)$ |
| 2025 | $(80,542,751)$ | - 0 - | 3,478,219 | $(84,020,970)$ |
| 2026 | $(89,668,826)$ | - 0 - | 872,171 | $(90,540,998)$ |
| 2027 | $(97,515,864)$ | -0- | 51,115 | $(97,566,979)$ |
| 2028 | $(105,138,177)$ | - 0 - | 27,188 | $(105,165,365)$ |
| 2029 | $(113,296,899)$ | - 0 - | 29,298 | $(113,326,197)$ |
| 2030 | $(121,330,729)$ | -0- | -0- | $(121,330,729)$ |

## VII. EFFECT OF FUTURE CONTRACT SALES

We have considered the effect of future contract sales on the existing Fund deficit. Our analysis assumes that contract sales resume for the 2007/08 enrollment period with contract payments beginning in February 2008. We examined three different levels of contract sales: 1,000 contracts each year; 2,000 contracts each year and 3,000 contracts each year. For each of these sales levels, we examined three different premium surcharge levels - 5.0\%, 7.5\% and 10.0\%

For each of these 9 scenarios, we projected future contract prices for each future projected enrollment period. We projected financial results for each future enrollment period according to the projected number of contracts and the amount of premium surcharge.

The number of future consecutive enrollment periods required to generate sufficient surplus to cure the existing deficit is shown in the table below.

| Enrollment Periods Required to Cure Deficit |  |  |  |
| :---: | :---: | :---: | :---: |
| Contracts Sold | 5\% Premium | 7.5\% Premium | 10\% Premium |
| 1,000 | 15 | 11 | 9 |
| 2,000 | 8 | 6 | 5 |
| 3,000 | 5 | 4 | 4 |

## VIII. SENSITIVITY TESTING

We believe that when there is a significant amount of uncertainty about conditions prevailing in the future it is important to test for adequacy under other possible assumptions.

We investigated the effect of variances in both university inflation and investment yield assumptions from those anticipated by the adequacy test assumptions. For these projections, we assumed no future contributions. These scenarios are described below. These scenarios are based on level adjustments to the baseline adequacy assumptions discussed earlier in this report.

1) Tuition inflation lower than adequacy test assumptions by $0.25 \%$ every year.
2) Tuition inflation higher than adequacy test assumptions by $0.25 \%$ every year.
3) Investment yields higher than adequacy test assumptions by $0.25 \%$ every year.
4) Investment yields lower than adequacy test assumptions by $0.25 \%$ every year.
5) Tuition inflation higher and investment yields lower than adequacy test assumptions by $0.25 \%$ every year.

The deficit for each of these scenarios is shown below.

| Sensitivity Testing Results |  |  |
| :---: | :---: | :---: | :---: |
| $\underline{\text { Scenario }}$ | $\underline{\text { Deficit }}$ | $\underline{\text { Change From Reported }}$ |
|  | $(\$ 17,573,641)$ | $\$ 2,735,597$ |
| 2 | $(\$ 23,111,505)$ | $(\$ 2,802,267)$ |
| 3 | $(\$ 17,595,703)$ | $\$ 2,713,535$ |
| 4 | $(\$ 23,101,614)$ | $(\$ 2,792,376)$ |
| 5 | $(\$ 25,979,015)$ | $(\$ 5,669,777)$ |

## IX. MONTE CARLO ANALYSIS

We have improved the Monte Carlo analysis of the Program in the following important areas:

- Incorporation of serial correlation,
- Incorporation of heteroscedasticity,
- Asset class returns treated as the risk-free return plus a spread and
- Incorporation of Bayesian techniques to better reflect experience.


## Serial Correlation

Serial correlation is the statistical connection of returns and inflation rates with prior returns and inflation rates. Many financial statistics show a strong relationship with their preceding values. For example, returns on Treasury Bills show a strong connection with returns for up to three years previously. Likewise, inflation at some of the University of Kentucky shows a connection with inflation for up to four years previously. We have constructed our Monte Carlo model to reflect these serial correlations.

## Heteroscedasticity

Heteroscedasticity is a technical term that means the volatility of a statistic changes over time. For those items in our projection that appear to have changing volatility, we have incorporated stochastic shifts in the volatility.

## Asset Class Returns Based on Risk-Free Return plus a Spread

Modern financial theory considers the risk-free return to be the fundamental component of the capital markets. Further, any investment can be considered as the sum of the risk-free return plus a spread reflective of the volatility of that investment's returns.

We have constructed our asset returns by modeling the 90-day Treasury Bill return as the risk-free return, then constructing separate models for equity spreads and fixedincome spreads.

## Bayesian Approach to Setting Parameters

We used a statistical technique known as "Bayesian statistics" to set the stochastic parameters in our model. Given prior beliefs regarding the stochastic elements in the projection, the Bayesian approach constructs the most-likely parameters for these stochastic elements based on historical information.

Not only does this improve the Monte Carlo model, it also allows the results of the Monte Carlo model to be used as a "yardstick" with which to judge the assumptions on which our actuarial reserve is based.

## Risk-Free Return Model

We modeled risk-free returns according to a lognormal distribution. Technically, we modeled the natural logarithm of the change in the risk free returns as a normal distribution. Modeling the natural logarithm as a normal distribution is exactly equivalent to modeling the underlying value as a lognormal distribution. The reason for using the change in returns rather than the returns is discussed below.

The autocorrelation (ACF) and partial autocorrelation (PACF) values of the natural log of the risk free returns are shown below. The dashed horizontal lines indicate the approximate $95 \%$ confidence interval for these values. The horizontal scale is the time lag. These values indicate that risk-free returns are highly autocorrelated and nonstationary. The standard approach for creating a stationary series is to take differences (that is, the value of the change rather than the underlying value).


We transformed the data into the changes and obtained the ACF and PACF shown below. From inspection, it is apparent that the changes are, if not stationary, at least close to stationary. It is also apparent that an autoregressive model with 2 or 3 factors would be appropriate in modeling the change in the natural log of the risk-free returns.


Our model for the change in the natural log of the risk free returns is:
$Y_{t}=\operatorname{Normal}\left(\mathrm{mu}_{\mathrm{t}}\right.$, sigmat $\left._{\mathrm{t}}\right)$
Where:
$Y_{t}$ is the change for year $t$
$\mathrm{mu}_{\mathrm{t}}=0.03538+0.2014\left(\mathrm{mu}_{\mathrm{t}-1}-.03538\right)-0.2869\left(\mathrm{mu}_{\mathrm{t}-2}-.03538\right)-0.1437\left(\mathrm{mu}_{\mathrm{t}-3}\right.$ - .03538)+Bernoulli(.03831) *Normal(0,.063) is the mean for year $t$

Bernoulli(.03831) is a Bernoulli distribution with a " p " of .03831
$\operatorname{Normal}(0, .063)$ is a Normal distribution with mean zero and standard deviation of . 063
sigma $_{t}=\operatorname{sigma}_{\mathrm{t}-1}$ * $\operatorname{Bernoulli}(.06608)$ * Trunc_Normal(1,2, $\left.0.2,5\right)$ is the variance for year t
Bernoulli(.06608) is a Bernoulli distribution with a "p" of . 06608
Trunc_Normal( $1,2,0.2,5$ ) is a truncated Normal distribution with mu $=1$, sigma $=2$
And truncated at 0.2 on the left and at 5 on the right.
$\operatorname{sigma}_{\mathrm{t}}=$ sigma $_{\mathrm{t}-1}$ if the Bernoulli distribution yields zero

In words, the mean is the sum of an autoregressive process plus an additive random shock. The standard deviation is subject to a random multiplicative shock. The Bernoulli factor for the mean results in a $3.8 \%$ likelihood of a shock in any year while the Bernoulli factor for the standard deviation gives a $6.6 \%$ likelihood of a change in the volatility in any year. The amount of the volatility change is proportional and varies from $1 / 5$ to 5 .

## Large Cap Equity Risk Premium

The chart below shows historic equity risk premiums for the S\&P 500 for the post-WWII era plotted against risk-free returns.


Equity premiums are stationary or close to stationary (in the statistical sense - that is, the average value doesn't move very much over time), so no differencing was applied to these rates. We modeled the geometric (as opposed to arithmetic) equity risk premium as follows:
$Z_{t}=\operatorname{Normal}\left(\mathrm{mu}_{\mathrm{t}}\right.$, sigma $)$
Where:
$\mathrm{Z}_{\mathrm{t}}$ is the risk-premium for year t
$\mathrm{mu}_{\mathrm{t}}=0.0631-1.988\left(\mathrm{Y}_{\mathrm{t}}-\mathrm{Y}_{\mathrm{t}-1}\right)-0.04906\left(\mathrm{mu}_{\mathrm{t}-3}-.0631\right)+0.2596\left(\mathrm{mu}_{\mathrm{t}-4}-.0631\right)-0.1215$

* $\mathrm{mu}_{\mathrm{t}-6}-.0631$ )
sigma $=.158$


## Small/Mid Cap Equity Risk Premium

The chart below shows historic equity risk premiums for the small cap for the postWWII era plotted against risk-free returns.


We have treated Small/Mid Cap equity risk premiums in a manner consistent with Large Cap equity risk premiums according to the following model
$X_{t}=\operatorname{Normal}\left(\mathrm{mu}_{\mathrm{t}}\right.$, sigma $)$
Where:
$X_{t}$ is the risk-premium for year $t$

```
\(\mathrm{mu}_{\mathrm{t}}=0.08368-2.819\left(\mathrm{Y}_{\mathrm{t}}-\mathrm{Y}_{\mathrm{t}-1}\right)-0.2045\left(\mathrm{mu}_{\mathrm{t}-1}-.08368\right)-0.1872\left(\mathrm{mu}_{\mathrm{t}-6}-.08368\right)-\)
    \(0.2045\left(\mathrm{mu}_{\mathrm{t}-7}-.08368\right)\)
    sigma \(=.168\)
```


## Fixed-Income Risk Premium

The chart below shows historic fixed-income risk premiums for the post-WWII era plotted against risk-free returns.


We have treated fixed-income risk premiums in a manner consistent with equity risk premiums according to the following model
$\mathrm{W}_{\mathrm{t}}=\operatorname{Normal}\left(\mathrm{mu}_{\mathrm{t}}\right.$, sigma $)$
Where:

$$
\begin{aligned}
& \mathrm{W}_{\mathrm{t}} \text { is the risk-premium for year } \mathrm{t} \\
& \mathrm{mu}_{\mathrm{t}}=0.01431-2.335\left(\mathrm{Y}_{\mathrm{t}}-\mathrm{Y}_{\mathrm{t}-1}\right)-0.09233\left(\mathrm{mu}_{\mathrm{t}-4}-.01431\right)-0.1805\left(\mathrm{mu}_{\mathrm{t}-5}-.01431\right)+ \\
& \quad 0.124\left(\mathrm{mu}_{\mathrm{t}-6}-.01431\right) \\
& \text { sigma }=
\end{aligned}
$$

## Final Fixed Income Returns

The fixed income risk premiums described above are based on Ibbotson's medium-term Treasury bond return data, while the Program actually invests in larger basket of fixed income securities. We selected the Lehman Brothers Government/Credit Index to represent the Program's fixed income returns - both Corporate and non-Corporate bonds. We performed regression analysis of the Index returns against Ibbotson's

Treasury returns in order to convert the Risk Free + Spread return to the benchmark return.

The regression equation is:

Benchmark Fixed Income Return $=.019881+.684845$ * Medium-term Treasury return.
The $r^{2}$ value from this regression is $94.2 \%$

## Non-U.S. Equity Risk Premium

The chart below shows historic fixed-income risk premiums for Morgan Stanley's EAFE returns (after exchange rates) for 1970-2005.


We have treated non-U.S. equity risk premiums in a manner consistent with equity risk premiums according to the following model
$\mathrm{V}_{\mathrm{t}}=\operatorname{Normal}\left(\mathrm{mu}_{\mathrm{t}}\right.$, sigma $)$
Where:
$\mathrm{V}_{\mathrm{t}}$ is the risk-premium for year t
$\mathrm{mu}_{\mathrm{t}}=0.06923-1.567\left(\mathrm{Y}_{\mathrm{t}-1}-\mathrm{Y}_{\mathrm{t}-2}\right)+0.1509\left(\mathrm{mu}_{\mathrm{t}-1}-.06923\right)-0.07435\left(\mathrm{mu}_{\mathrm{t}-2}-.06923\right)-$ $0.2999\left(\mathrm{mu}_{\mathrm{t}-4}-.01431\right)$
sigma $=.213$

## Tuition Inflation

We modeled tuition inflation as a Beta distribution with varying parameters. That is,
$\mathrm{U}_{\mathrm{t}}=\operatorname{Beta}\left(\right.$ alpha $_{\mathrm{t}}$, beta $\left._{\mathrm{t}}\right)$
Where:
$U_{t}$ is the tuition inflation for year $t$
alphat $_{t}$ and beta ${ }_{t}$ are the Beta distribution parameters for year $t$, and are determined in the standard manner from year t's mean and variance.
mean $_{t}=0.07783+0.2007\left(\right.$ mean $\left._{t-1}-.07783\right)-0.1957\left(\right.$ mean $\left._{t-4}-.07783\right)$ + Bernoulli(.1207) * Normal(0, .0109)
Bernoulli(.1207) is a Bernoulli distribution with a "p" of . 1207
$\operatorname{Normal}(0, .0109)$ is a Normal distribution with mean zero and standard deviation of 0.0109
variance $_{t}=$ variance $_{t-1}$ * Bernoulli(.0934) * Truncated-Normal(1,2, 0.2,5) if the Bernoulli distribution generates unity.
variance $_{\mathrm{t}}=$ variance $_{\mathrm{t}-1}$ if the Bernoulli distribution generates zero.
Bernoulli(.0934) is a Bernoulli distribution with a "p" of . 0934
Truncated-Normal $(1,2,0.2,5)$ is a truncated normal distribution with $\mathrm{mu}=1$, sigma $=2$, truncated on the left at 0.2 and on the right at 5.0.

## Results

These results are summarized in the table below and shown graphically in the chart following the table.

| Proportion of Projections With a Surplus | $50.9 \%$ |  |
| :--- | ---: | ---: |
| $25 \%$ of results are better than: | $\$ 23,047,800$ | Surplus |
| $50 \%$ of results are better than: | $\$ 1,169,486$ | Surplus |
| $75 \%$ of results are better than: | $(\$ 49,951,876)$ | Deficit |
| Best Result | $\$ 43,876,236$ | Surplus |
| Worst Result | $(\$ 372,808,096)$ | Deficit |
| Mean Result |  | Deficit |

Distribution for Surplus/R9

$64 \%$ of the results are better than the reported deficit of $(\$ 20,309,238)$. The $50 \%$ Results measure is a "best-estimate" measure of results. If our assumptions are neither conservative (that is they understate results) nor aggressive (that is they overstate results) then the $50 \%$ Results measure should be close to our projected result of $(\$ 20,309,238)$. The table above indicates that our assumptions contain a significant degree of conservative.

The Worst Result indicates what happens if economic events continue adversely for the lifetime of the current units - high tuition increases, coupled with negative returns in the equity market until the end of the projection horizon. On the other hand, the Best Result indicates what happens if economic conditions are favorable for the remaining lifetime of the current units.

## Commentary on the Results of the Stochastic Analysis

The assumptions for the reported result were based on expert opinion, while the stochastic analysis used statistical parameters derived from Bayesian "look-back" methodologies that use historical results as a basis.

Expert opinion, whether consciously or unconsciously, gives more weight to recent experience than to less-recent experience. This can be seen especially in the assumptions for investment returns and in the inflation assumptions.

Our inflation assumptions are based on the rationale that the recent run-up in tuition will continue for a couple of years, due to the observation that tuition increases at the University of Kentucky have almost no correlation with legislative appropriations. In the Bayesian model used in the Monte Carlo analysis, inflation rates moderate in the near future before moving back to the historical trend.

Likewise, the stochastic model's investment returns are higher than those based on expert opinion.

The $7.76 \%$ investment return assumption is based on the expert opinion from the Program's investment advisor, Evaluation Associates. This expert opinion is consistent with what we have seen for similar programs with similar asset portfolios, and implies an equity risk premium lower than historical averages. The stochastic model is based on historical equity premiums, so will produce results consistent with historical equity premiums.

Many investment experts believe that equity risk premiums in the future will be consistently lower than historical averages - perhaps influenced by equity returns of the early 2000's. However, such expert opinion regarding equity risk premiums is not unanimous. We offer the following quotes from experts in defense of historically consistent equity premiums:

Brealey and Myers have no official position on the exact market risk premium, but we believe a range of 6 to $8.5 \%$ is reasonable for the United States. We are most comfortable with figures towards the upper end of the range ${ }^{3}$

Note that this quote is from what is perhaps the most widely used undergraduate text on finance, written by two respected economists.

The next quote is taken from the actuarial literature

It is dangerous for actuaries to engage in simplistic analyses of historical ERPs to generate ex ante forecasts that differ from the realized mean. ${ }^{4}$

In other words, any projection of equity returns that deviates from historical risk premiums needs to have a compelling reason for such a departure. Without such a compelling reason, historical norms should be used.

[^1]We also note that there is consistency between the development of asset returns and inflation rates since both are developed from long-term history. This consistency is important since the spread between investment returns and inflation is more important than the absolute level of either. In this sense, the stochastic projections provide evidence that the spread used in the deterministic projection is reasonable to conservative when considered in the aggregate.

## X. CHANGES IN ACTUARIAL ASSUMPTIONS

We made three changes to the assumptions used in projecting the actuarial deficit. These assumptions changes are, in aggregate, conservative - that is, they cause the deficit to be larger than it would have been without these changes. These changes are discussed below.

## Changes in Expenses

We updated the assumption for aggregate expenses to reflect the current budget of the program as shown below.

| Current Assumption | Prior Assumption |
| :---: | ---: |
| $\$ 647,397$ | $\$ 840,184$ |

We also updated the assumptions for investment expenses to reflect the Program's revised contractual expenses for its asset manager.

| Current Assumption | Prior Assumption |
| :--- | :---: |
| 35 basis points on all assets | 49 basis points in the $1^{\text {st }} \$ 25,000,000$ |
|  | 28 basis points in the next $\$ 25,000,000$ |
|  | 21 basis points on excess over $\$ 50,000,000$ |

## Change in Tuition Inflation

We revised the tuition inflation assumptions to better reflect our long-term view of what tuition increases will be.

| Current Assumption | Prior Assumption |
| :---: | :---: |
| $11.00 \%$ for $2007 / 08$ | $7.50 \%$ through $2011 / 12$ |
| $10.00 \%$ for $2008 / 09$ | $7.25 \%$ though $2013 / 14$ |
| $7.00 \%$ thereafter | $7.00 \%$ thereafter |

## Dollar Effect of Change in Assumptions

If assumptions had been the same as last year, the Program's deficit would have been:

These three changes increased the deficit by $\$ 5,806,459$. The effect of the inflation assumption change by itself was to increase the deficit by $\$ 6,101,895$.

## XI. EXPECTED USE OF FUNDS

The Fund, which is comprised of contributions, fees, all interest and earnings, and any other money appropriated or made available to KAPT, is expected to pay benefits and expenses in the following proportions:

- Tuition payments - $94.2 \%$
- Expenses - 3.3\%
- Payments of refunds to contract owners - $2.5 \%$

These results are shown graphically below.

## Expected Use of KAPT Funds




[^0]:    ${ }^{1}$ The projected increase represents interest on the beginning deficit amount, plus some additional amounts due to the change in the non-level tuition inflation assumptions.
    ${ }^{2}$ All Other is comprised mainly of differences between projected and actual expenses and of differences between projected and actual contract cancellations.

[^1]:    ${ }^{3}$ Brealey \& Myers in Principles of Corporate Finance, 6 th edition, page 160, McGraw-Hill, 2000
    ${ }^{4}$ Derrig \& Moore, "Equity Risk Premium: Expectations Great and Small", North American Actuarial Journal, Volume 8, Number 1, page 60.

